Using Binomial Theorem to get a Taylor Series

For large N, $e \approx \left(1+\frac{1}{N}\right)^N$

Binomial Theorem: $(a+b)^N = \sum_{n=0}^{N} \binom{N}{n} a^n b^{N-n} = \sum_{n=0}^{N} \frac{N!}{n!(N-n)!} a^n b^{N-n}$

$$e \approx \left(1+\frac{1}{N}\right)^N = \sum_{n=0}^{N} \frac{1}{n!} \frac{N!}{(N-n)!} \left(\frac{1}{N}\right)^n = \sum_{n=0}^{N} \frac{1}{n!} \left(\frac{\prod_{i=0}^{n-1}(N-i)}{N^n} \right)$$

If $N \gg n, i$, then $\prod_{i=0}^{n-1}(N-i) \approx N^n$,

so $\frac{\prod_{i=0}^{n-1}(N-i)}{N^n} \approx 1$

$$\boxed{e \approx \left(1+\frac{1}{N}\right)^N \approx \sum_{n=0}^{N} \frac{1}{n!}}$$ Taylor Series

What about the Taylor Series for e^x, where x is small?

$e^x \approx \left(1+\frac{1}{N}\right)^{Nx}$ Let's set $x = \frac{1}{N-m}$, where $N \gg m$. When $m=0$, you instantly get the well known approximation $e^x \approx 1+x$. From what I know, $\lim_{x\to 0} e^x$ and $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$, it's ok to have these limits be the same/similar/related rates, so a function where $x = f(N)$ where $N \to \infty$ and $x \to 0$ is ok.

Apply Binomial Theorem again.

$$e^x \approx \left(1+\frac{1}{N}\right)^{N/N-m} = \sum_{n=0}^{N} \frac{1}{n!} \left(\frac{\frac{N}{N-m}!}{\left(\frac{N}{N-m}-n\right)!} \right) \left(\frac{1}{N}\right)^n = \sum_{n=0}^{N} \frac{1}{n!} \left(\frac{\prod_{i=0}^{n-1}\left(\frac{N}{N-m}-i\right)}{N^n} \right)$$

If $N \gg m, n, i$, then $\prod_{i=0}^{n-1}\left(\frac{N}{N-m}-i\right) \approx \frac{N^n}{(N-m)^n}$

$$= \sum_{n=0}^{N} \frac{1}{n!} \left(\frac{1}{N-m}\right)^n = \boxed{\sum_{n=0}^{N} \frac{x^n}{n!}}$$ Taylor Series